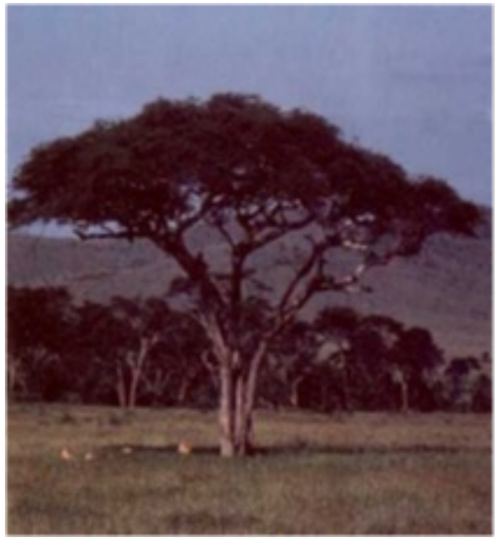
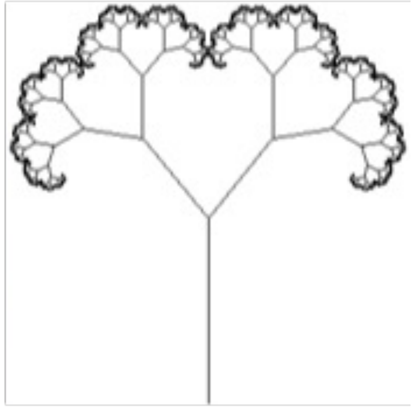
Water Resources Engineering and Management

(CIVIL-466, A.Y. 2024-2025)

5 ETCS, Master course





Lecture 8-3: Optimisation and linear programming

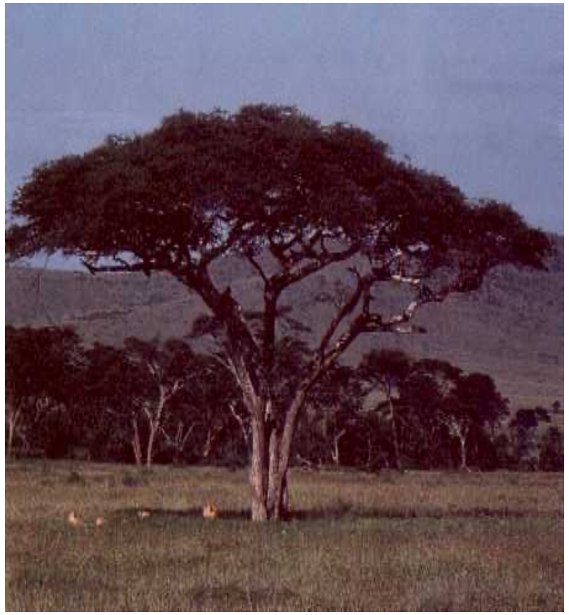
Prof. P. Perona

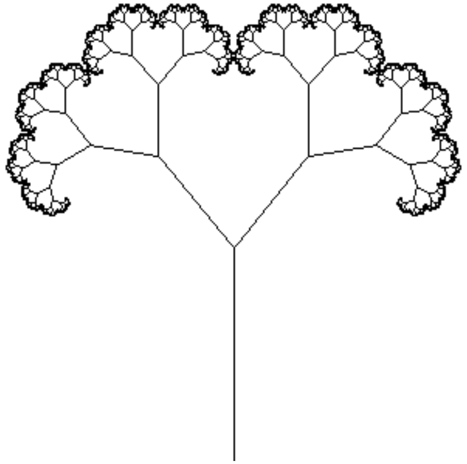
Platform of hydraulic constructions

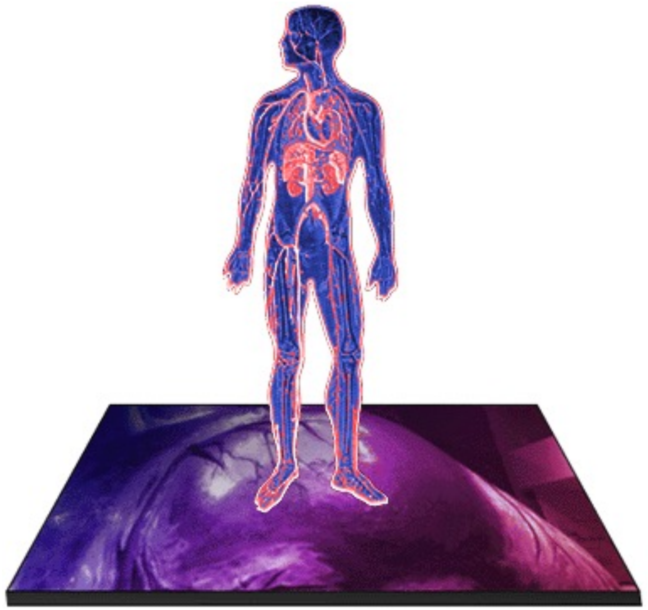
PLATEFORME DE CONSTRUCTIONS HYDRAULIQUES

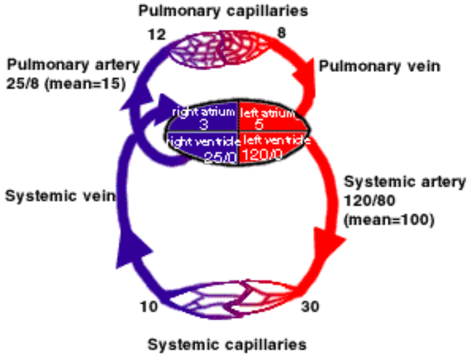
EPFL

Optimization problems in nature





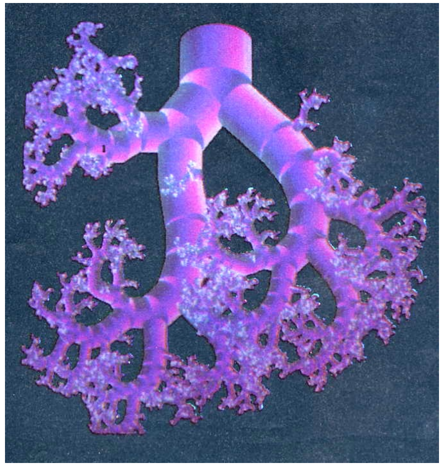


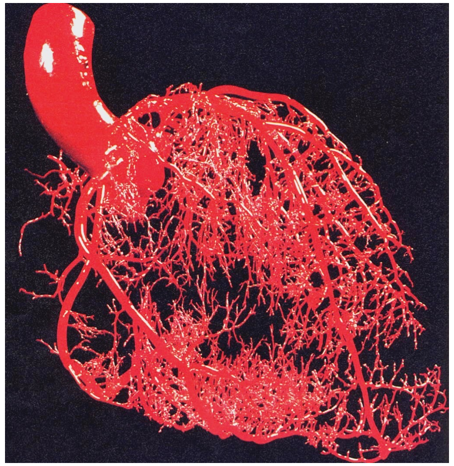


Typical fractal structures in vegetation canopy and…

Problem: for respiratin you need to generate a geometrical object having the max perimeter/surface for exchange processes but staying within a finite surface/volume (e.g. body)

In the human body

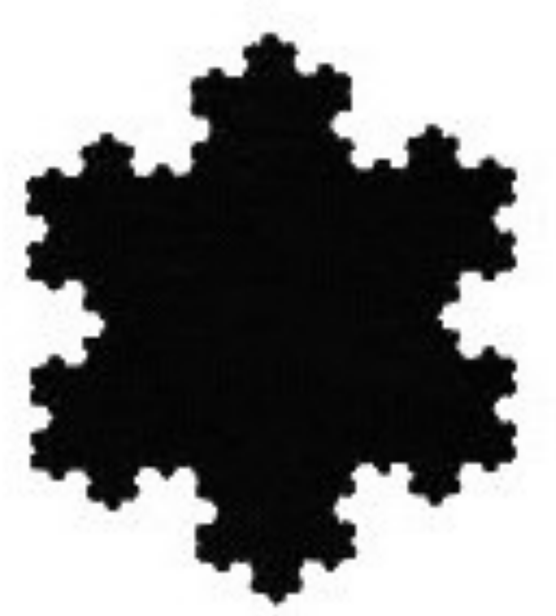




Optimization problems in nature

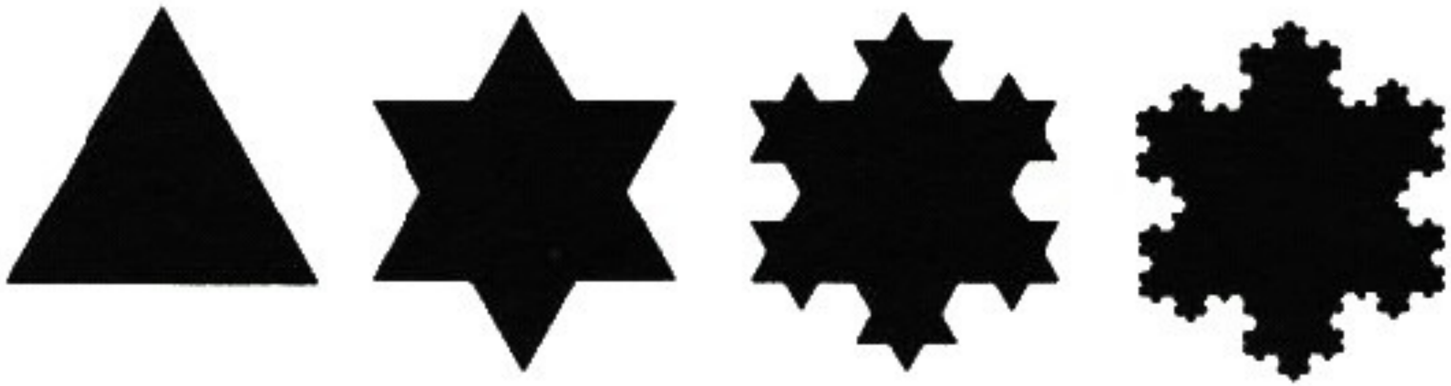
Objective functions in nature are not targeting economical benefit, rather biological ones

Von Kock curve



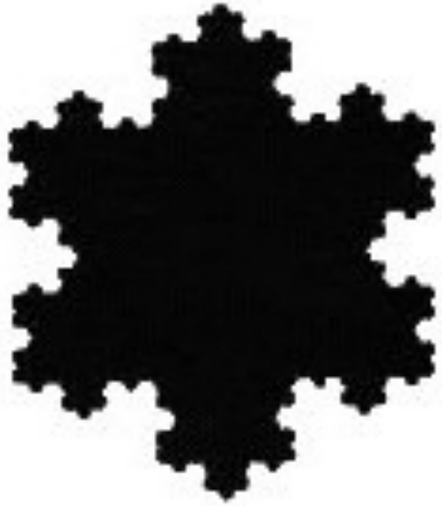
Optimization problem in nature

Von Kock curve



Area is Finite, the Perimeter becomes infinite!!

Try to prove it!



Water resources engineering models

THREE TYPES:

• Simulation models. Are a conceptualization of a water system and are used to predict its hydrological response under predefined operational conditions

• Optimization models. Are able to take a decision (i.e., allocate water) depending on water inflow and demand. They automatically search for an optimal solution to the water allocation problem for all the time periods of interests, e.g. allow for dynamic programming.

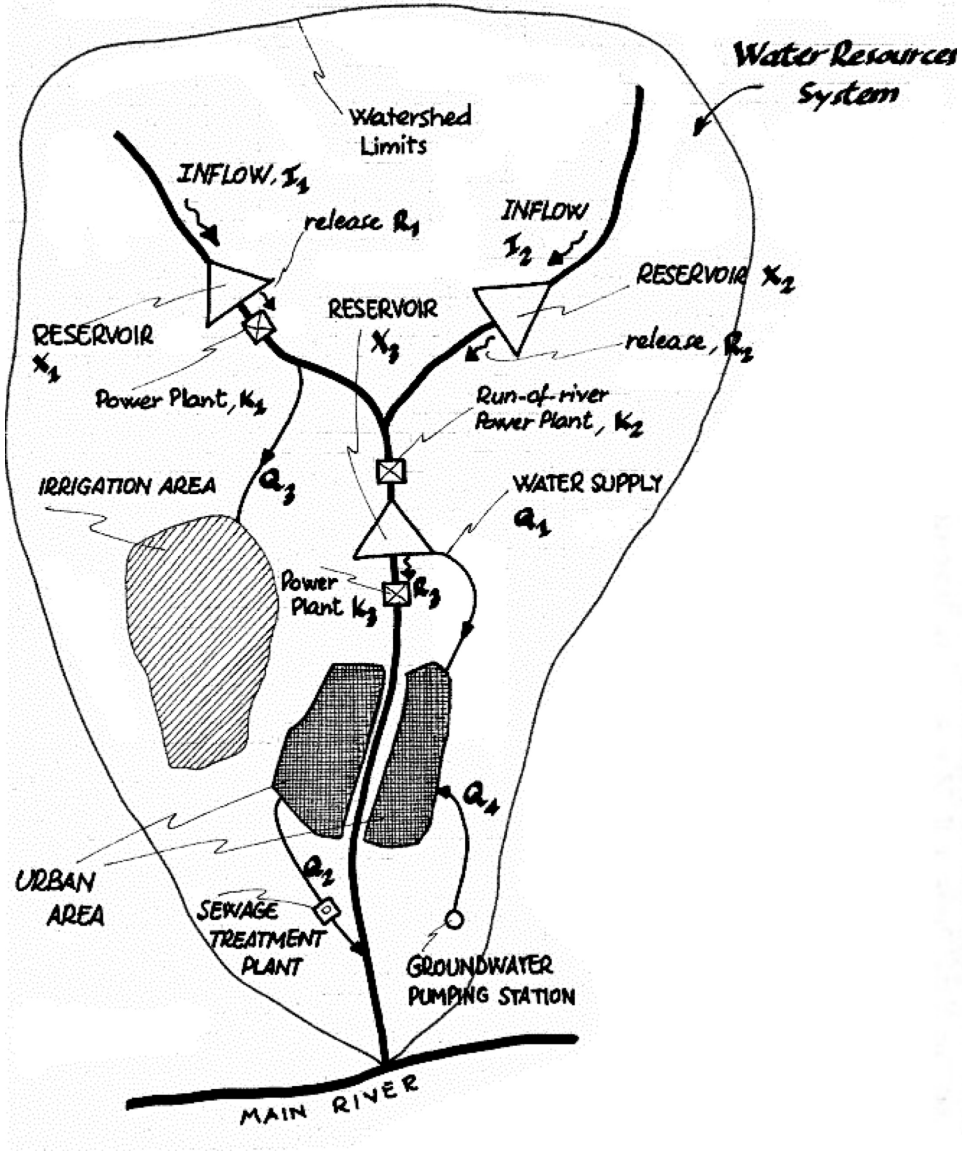
• Network-flow models. Basically, these models can combine characteristics of both simulation and optimization but their performance are limited and does not allow for dynamic programming.

Which types of water resources engineering models?

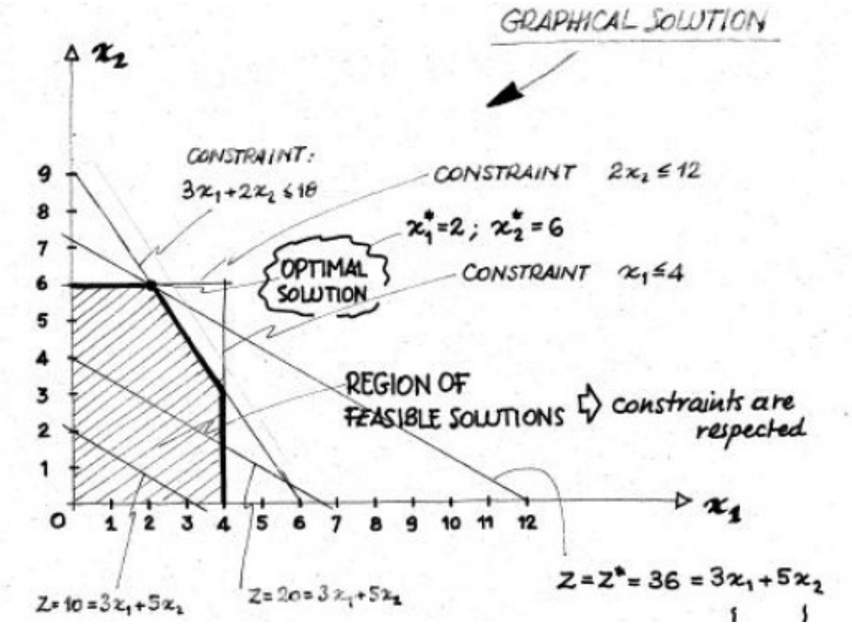
Other examples:
REsources ALlocation Model (REALM)
Perera et al., J. Env. Manag. 2006

SWAM (Simplified water Allocation model)
https://hydrology.dnr.sc.gov/pdfs/swm/TechnicalReports/SWAMusermanv4dot0.pdf

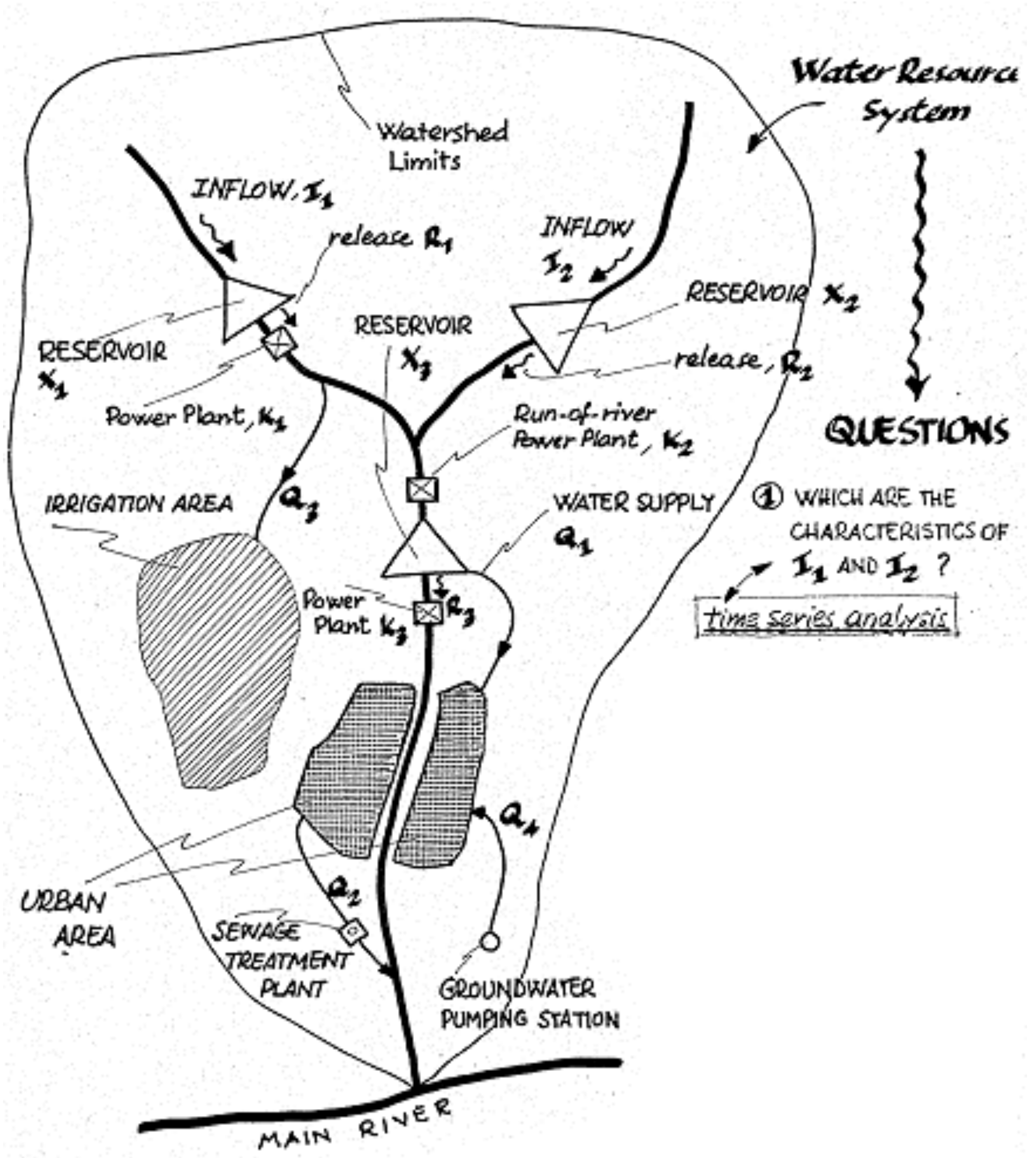
Some typical questions



What is the optimal allocation of water resource among multiple uses (e.g. multiple crops)?



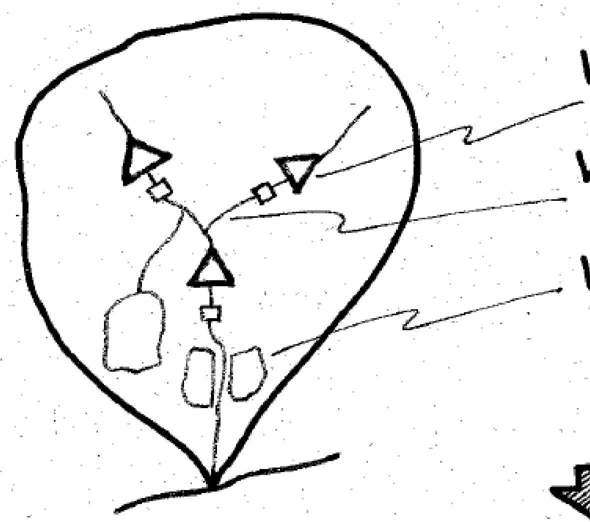
The river basin system revisited



P. Perona, P. Burlando - ETH Zurich

Courtesy of Prof. P. Burlando

WATER RESOURCES ENGINEERING



WHICH CAPACITIES OF RESERVOIRS ?

WHICH RELEASES FROM RESERVOIRS ?

WHICH CONSTRAINTS, e.g. WATER SUPPLY DEMAND, IRRIGATION REQUIREMENTS, HYDROPOWER, ...

mathematical model of the system

DESIGNED TO LOOK FOR AN OPTIMAL SOLUTION

ONE SHOULD DEFINE

- THE **decision variables**, I.E. THE VARIABLES CONTROLLING THE SYSTEM DYNAMIC

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- THE **objective function**, WHICH DESCRIBES THE PERFORMANCE CRITERIA OF THE SYSTEM
- THE **constraints**, WHICH DESCRIBE PHYSICAL, ECONOMIC LIMITS OF THE SYSTEM.

AND SOLVE THE MATHEMATICAL PROBLEM, SEARCHING FOR AN

optimal solution, I.E. A SET OF VALUES OF THE DECISION VARIABLES THAT SATISFIES THE CONSTRAINTS AND PROVIDES AN OPTIMAL VALUE OF THE OBJECTIVE FUNCTION

The methods suitable for solving the mathematical problem are function of the system characteristics

e.g. linear programming vs. nonlinear

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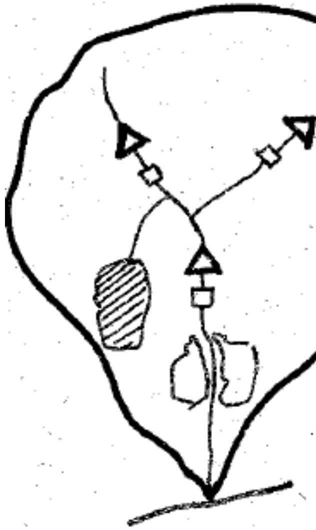
example of system definition



decision variables

X_1, X_2, X_3 RESERVOIR CAPACITIES

R_1, R_2, R_3 RELEASES FROM RESERVOIRS

S_1, S_2, S_3 STORAGE

Q_i FLOWS OF DIFFERENT SYSTEM COMPONENTS

K_i CAPACITIES OF HYDROELECTRIC POWER PLANTS

....

... ...

$R_1 = f(S_1, I_1)$

I_1 — INFLOW

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objective function, Z

$$\max Z = b_1(R_1, K_1) + b_3(R_3, K_3) + b_4(Q_1) + c_1(X_1) - C_2(X_2) - C_3(X_3)$$

- $b_1(R_1, K_1) + b_3(R_3, K_3)$: BENEFITS FROM ENERGY PRODUCTION AS FUNCTION OF RELEASE AND CAPACITY OF POWER PLANT
- $b_4(Q_1)$: BENEFITS FROM WATER SUPPLY
- $c_1(X_1) - C_2(X_2) - C_3(X_3)$: CONSTRUCTION COSTS OF RESERVOIRS

constraints

$X_1 < X_1^*$, I.E. MAXIMUM CAPACITY OF RESERVOIR 1, e.g. DUE TO GEOLOGIC CONSTRAINTS

$R_3 \geq R_3^*$, I.E. MINIMUM RELEASE DUE TO ECOLOGICAL CONSTRAINTS

$E_1 \geq E_1^*$, I.E. MINIMUM ENERGY PRODUCTION

... ...

... ...

⇨ IN GENERAL

$$\max Z = f(x_1, x_2, \dots x_N)$$

$$g(x_1, x_2, \dots x_N) \gtreqless b_i \qquad i = 1, \dots, M$$

($x_1, x_2, \dots x_N$: decision variables; b_i: constraints)

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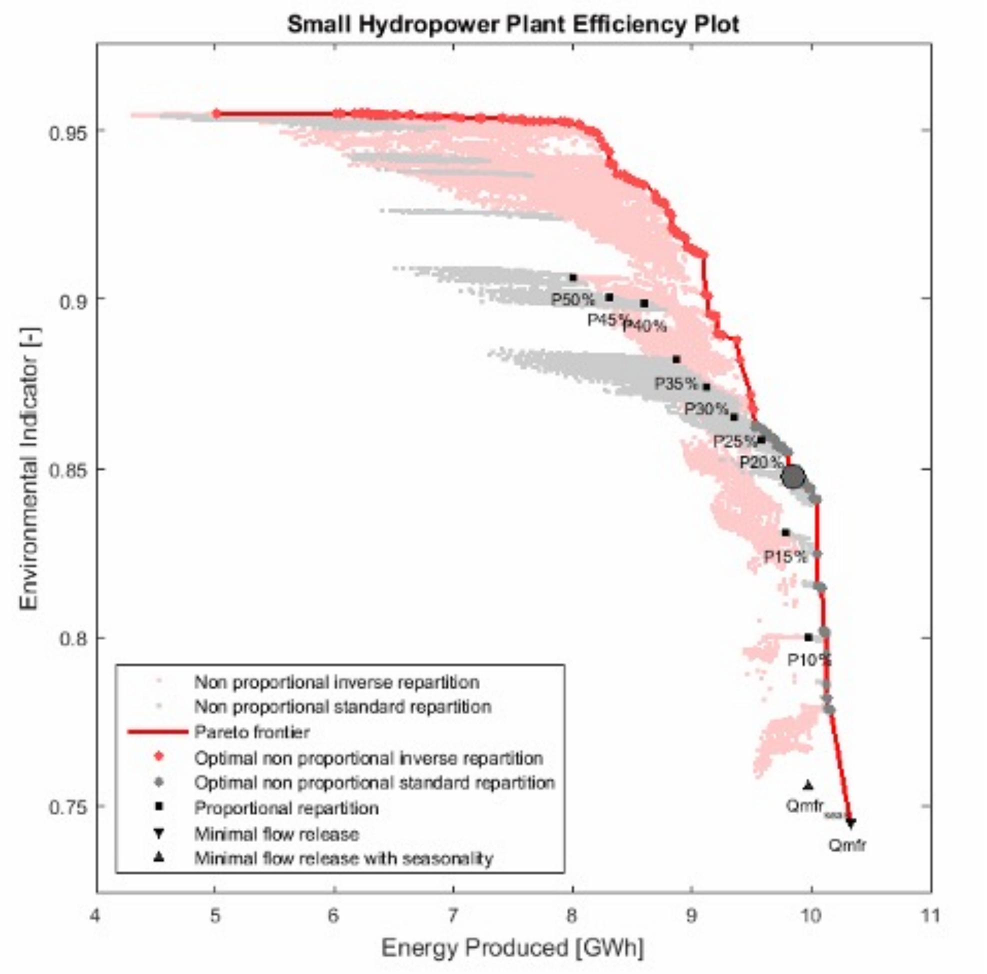
Definition: the Pareto optimum

Given a set of alternative allocations of, say, goods or income for a set of individuals, a change from one allocation to another that can make at least one individual better off without making any other individual worse off is called a **Pareto improvement**. An allocation is **Pareto efficient** or **Pareto optimal** when no further Pareto improvements can be made. This is often called a **strong Pareto optimum (SPO)**. The topological location where such situation occurs is called a **Pareto frontier**



Linear Programming - LP

LINEAR PROGRAMMING IS A MATHEMATICAL APPROACH TO OPTIMAL SOLUTION OF PROBLEMS BASED ON THE *linearity* OF THE *objective function* AND *constraints*

$$\max Z = c_1 x_1 + c_2 x_2 + \dots + c_n x_n = \sum_{1}^{n} c_j x_j$$

$$\left.\begin{aligned} a_{11} x_1 + a_{12} x_2 + \dots + a_{1n} x_n &\leq b_1 \\ a_{21} x_1 + a_{22} x_2 + \dots + a_{2n} x_n &\leq b_2 \\ &\vdots \\ a_{m1} x_1 + a_{m2} x_2 + \dots + a_{mn} x_n &\leq b_m \end{aligned}\right\} \quad \sum_{1}^{n} a_{ij} x_j = b_i \qquad i = 1, \dots, m$$

$$x_j \geq 0 \qquad j = 1, \dots n$$

LP IS CHARACTERISED BY THE ASSUMPTIONS OF

proportionality: THE CONTRIBUTION OF THE j-TH DECISION VARIABLE TO THE SYSTEM EFFECTIVENESS $c_j x_j$ AND ITS USAGE OF THE VARIOUS RESOURCES $a_{ij} x_j$ ARE $\propto$ TO THE VALUE OF x_j

additivity (INDEPENDENCY): NO INFLUENCE OF ACTIVITY x_j ON x_i, THAT IS THE OVERALL CONTRIBUTION OF x_j AND x_i IS EQUAL TO THE SUM OF THE SINGLE ACTIVITIES

divisibility: NON INTEGER VALUES OF THE DECISION VARIABLES ARE PERMISSIBLE

deterministic: NO UNCERTAINTY CHARACTERISES MODEL'S PARAMETER (c_j, b_i, a_{ij}). UNCERTAINTY IS ACCOUNTED FOR BY SENSITIVITY ANALYSIS

Linear Programming – example

2-DIMENSIONAL CASE

$$\max Z = 3x_1 + 5x_2$$

⇦ OBJECTIVE FUNCTION

ⓐ $x_1 \leq 4$ ⓒ $3x_1 + 2x_2 \leq 18$

ⓑ $2x_2 \leq 12$ ⓓ $x_1 \geq 0 \quad x_2 \geq 0$

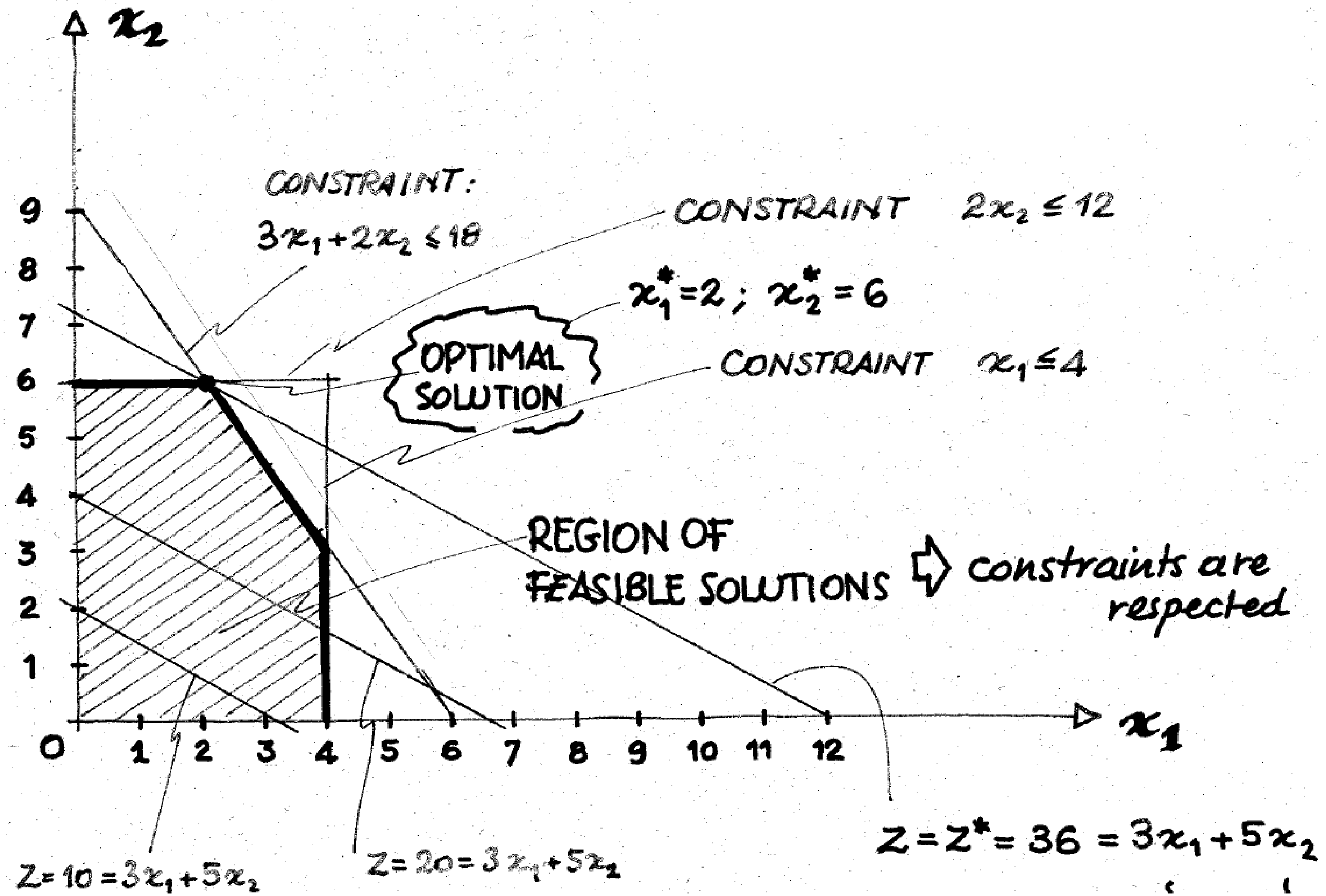
⇦ CONSTRAINTS

Graphical solution

THE SOLUTION

- LIES ON THE BORDER OF THE REGION OF FEASIBLE SOLUTIONS
- LIES AT THE INTERSECTION OF $n=2$ CONSTRAINT EQUALITIES, I.E. ⓑ and ⓒ. ALL OTHER CONSTRAINTS ARE INACTIVE

↓ $n-m$ ↓ IMPLICITELY VERIFIED



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Linear Programming - IRRIGATION EXAMPLE

PLATEFORME DE CONSTRUCTIONS HYDRAULIQUES

DATA: WATER AVAILABILITY, LAND EXTENSION, CROP PRODUCTIVITY, MANPOWER, SPECIFIC BENEFIT

PROBLEM: TO DETERMINE THE AMOUNT OF EACH CROP THAT MAXIMIZES THE BENEFITS

RESOURCE INPUT	MAXIMUM AVAILABLE INPUT	x_1 CROP 1	x_2 CROP 2
Land	50 ha	2 ha/ton	3 ha/ton
Water	$250 \cdot 10^6$ m^3	20 $10^6 m^3$/ton	5 $10^6 m^3$/ton
Manpower	90 Man Months	6 MM/ton	4 MM/ton
benefit	max ?	18 $/ton	21 $/ton

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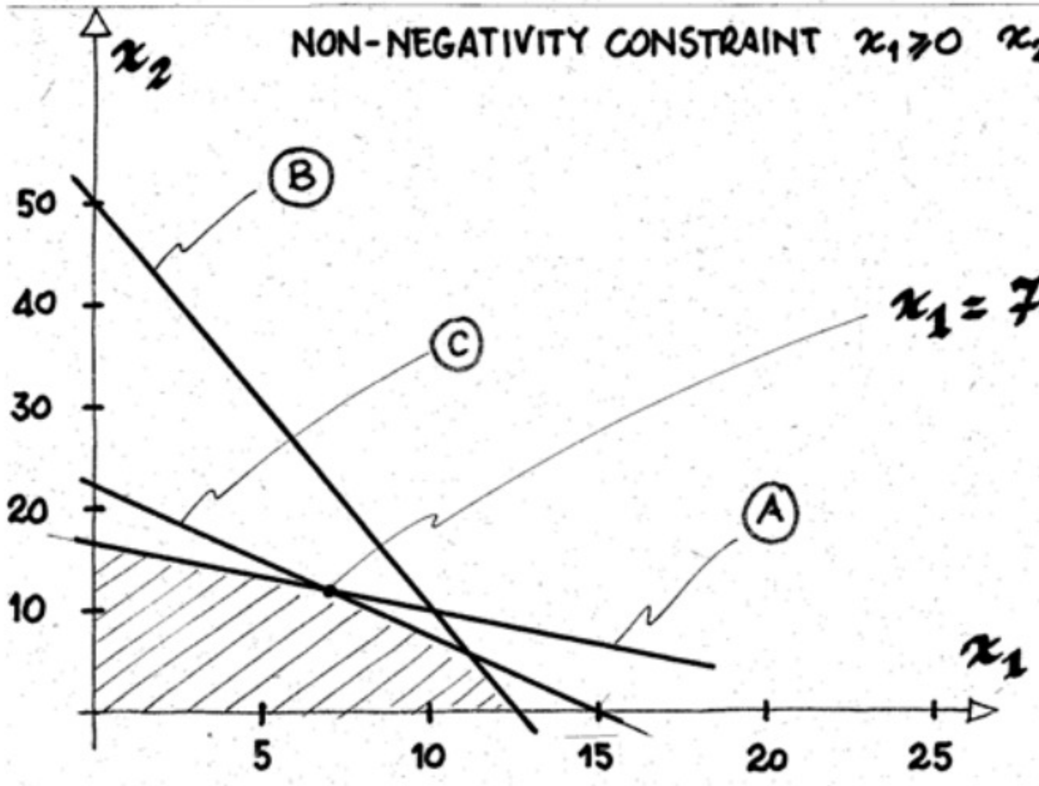
OBJECTIVE FUNCTION ⇨ $\max Z = 18x_1 + 21x_2$

CONSTRAINTS

$2x_1 + 3x_2 \leq 50$	Land constraint	(A)
$20x_1 + 5x_2 \leq 250$	Water constraint	(B)
$6x_1 + 4x_2 \leq 90$	Manpower constraint	(C)

NON-NEGATIVITY CONSTRAINT $x_1 \geq 0$ $x_2 \geq 0$



$x_1 = 7$; $x_2 = 12$

⇓

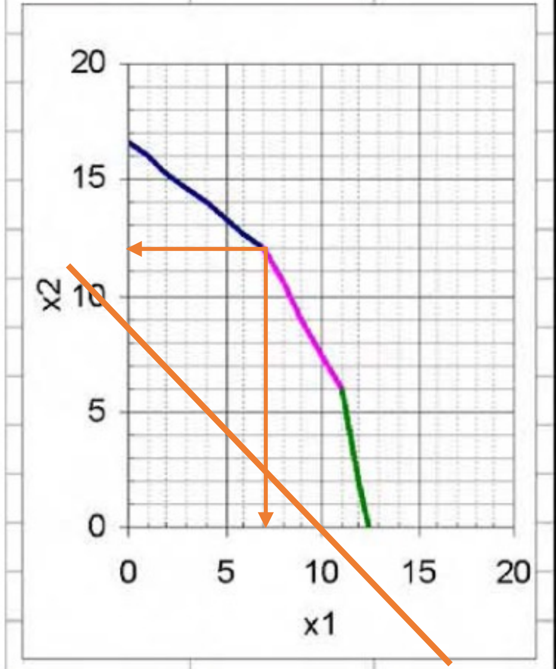
$z = 378$

(A) ⇨ ✓

(B) ⇨ $200 \leq 250$!

(C) ⇨ ✓

	Solver	
	max Z =	378
x_1 =	7	
x_2 =	12	
A =	50	<=50
B =	200	<=250
C =	90	<=90



	A	B	C			
	A<=50	B<=250	C<=90	Z	Z	Z
0	16.67			350		
1	16.00			354		
2	15.33			358		
3	14.67			362		
4	14.00			366		
5	13.33			370		
6	12.67			374		
7	12.00		12.00	378		378
8			10.50			364.5
9			9.00			351
10			7.50			337.5
11		6.00	6.00		324	324
12		2.00			258	
13		-2.00			192	
14		-6.00			126	
15		-10.00			60	
16		-14.00			-6	
17		-18.00			-72	
18		-22.00			-138	
19		-26.00			-204	
20		-30.00			-270	

Take home messages from these three lectures

- L8.1 I can list and explain the main environmental issues related to dam construction and operation
- L8.1 I know how to calculate the Q_{347} minimal flow reference and explain proportional and non-proportional policies
- L8.1 I can explain concerns about greenhous gas emissions from reservoirs
- L8.1 I know the main steps of EIA procedure and illustrate each of their scopes
- L8.1 I can explain how to build an Interaction matrix linking actions and environmental components

- L8.2 I understand the difference between zero sum and non-zero sum games
- L8.2 I can explain the prisoner's dilemma and build the corresponding payoff matrix
- L8.2 I can explain the difference between cooperative (win-win) and non-cooperative (lose-lose) solutions as well as win-lose ones
- L8.2 I can build the payoff matrix for a simple groundwater pumping transboundary problem

- L8.3 I can explain the mathematical set of equations and topology of a general optimisation problem
- L8.3 I understand the concept of Pareto optimum and Pareto frontier
- L8.3 I can write the basic equations regulating linear programming technique
- L8.3 I know how to solve a simple 2D optimisation problem with LP
- L8.3 I can explain the graphical meaning of the objective function and find the solution to a 2D LP in the decision variables space